Today you will:

- Write functions representing translations and reflections
- Write functions representing combinations of transformations
- Practice using English to describe math processes and equations

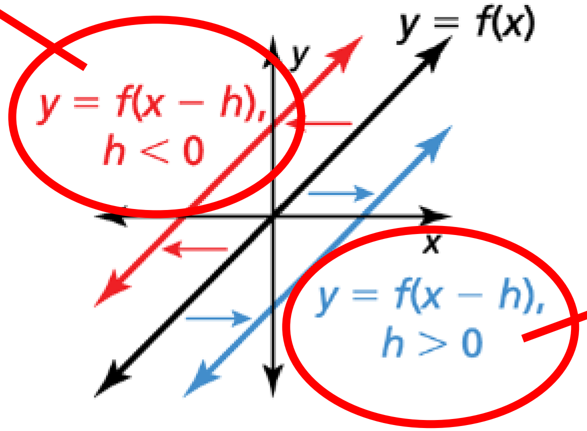
Core vocabulary:

- Parent function – most basic function for a given family of functions
- Transformation (of a graph of a function) – change a function's position, size, shape, orientation
- Translation (of a graph of a function) – shift a function horizontally and/or vertically
- Reflection (of a graph of a function) – flip a function around a given line (line of reflection)

Horizontal translation

- The graph of $y = f(x - h)$ is a horizontal translation of the parent function $y = f(x)$ where $h \neq 0$.

Example of horizontal translation left 3
$f(x) = (x + 3)^2$



Example of horizontal translation right 3:
$f(x) = (x - 3)^2$

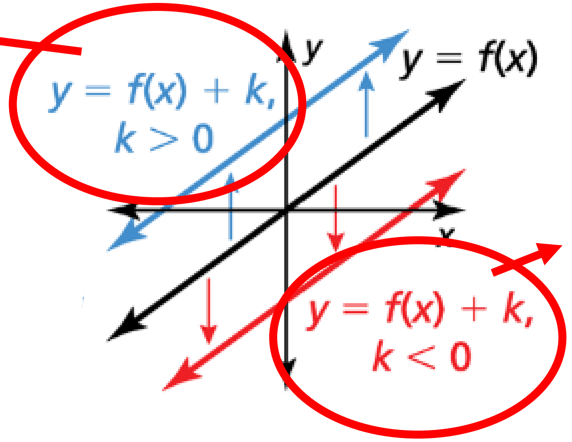
Horizontal translation is *INSIDE* the parenthesis with the x

Seems backwards but
subtracting moves right
adding moves left.

Vertical translation

- The graph of $y = f(x) + k$ is a vertical translation of the parent function where $k \neq 0$.

Example of vertical translation up 3:
$f(x) = x + 3$



Example of vertical translation down 3:
$f(x) = x - 3$

Vertical translation is *OUTSIDE* the parenthesis, apart from the x

Adding moves up
Subtracting moves down

Some translation examples:

- $f(x) = (x-4)^2 + 2$
  - Translation 4 units right, 2 units up
- $f(x) = (x+5)^2 - 3$
  - Translation 5 units left, 3 units down
- $f(x) = (x-1)^2$
  - Translation 1 unit right (no vertical translation)
- $f(x) = x^2 + 1$
  - Translation 1 unit up (no horizontal translation)

EXAMPLE 1: Let $f(x) = 2x + 1$.

a. Write a function g whose graph is a translation 3 units down of the graph of f.

b. Write a function h whose graph is a translation 2 units to the left of the graph of f.

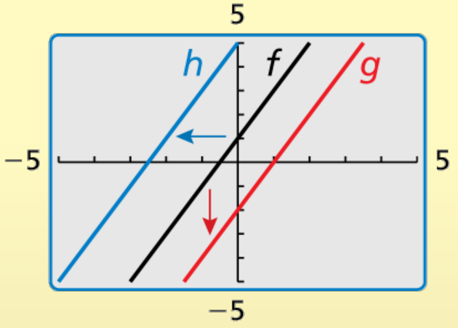
SOLUTION

a. A translation 3 units down is a vertical translation that adds -3 to each output value.

$g(x) = f(x) + (-3)$	Add -3 to the output.
$= 2x + 1 + (-3)$	Substitute $2x + 1$ for $f(x)$.
$= 2x - 2$	Simplify.

The translated function is $g(x) = 2x - 2$.

Check



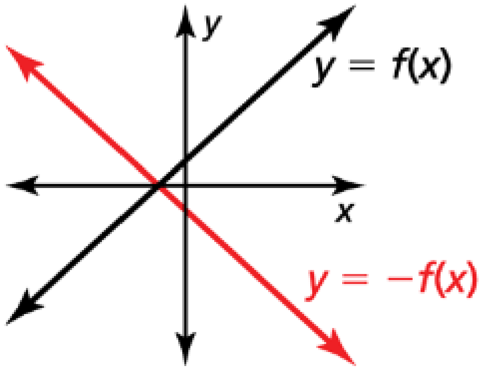
b. A translation 2 units to the left is a horizontal translation that subtracts -2 from each input value.

$h(x) = f(x - (-2))$	Subtract -2 from the input.
$= f(x + 2)$	Add the opposite.
$= 2(x + 2) + 1$	Replace x with $x + 2$ in $f(x)$.
$= 2x + 5$	Simplify.

The translated function is $h(x) = 2x + 5$.

Reflections in the x-axis

- The graph of $y = -f(x)$ is a reflection in the x-axis of the graph of $y = f(x)$.



Example of a reflection around the x-axis:
$f(x) = -2x$

is a reflection of $f(x) = 2x$ around the x-axis

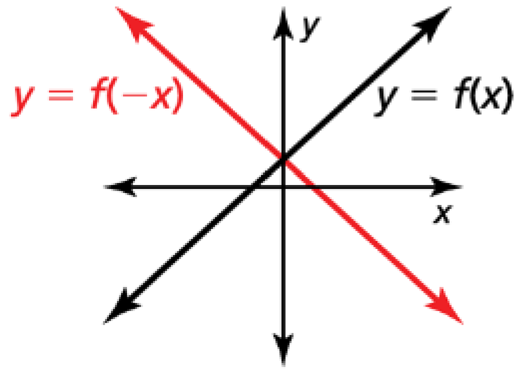
*Multiplying the **outputs** by -1 changes their signs and flips the graph around the x-axis*

Reflections in the y-axis

- The graph of $y = f(-x)$ is a reflection in the y-axis of the graph of $y = f(x)$.

Example of a reflection around the y-axis:
$f(x) = -2x + 3$

is a reflection of $f(x) = 2x + 3$ around the y-axis



*Multiplying the **inputs** by -1 changes their signs and flips the graph around the y-axis*

EXAMPLE 2: Let $f(x) = |x + 3| + 1$.

a. Write a function g whose graph is a reflection in the x-axis of the graph of f.

b. Write a function h whose graph is a reflection in the y-axis of the graph of f.

SOLUTION

a. A reflection in the x-axis changes the sign of each output value.

$g(x) = -f(x)$ — Multiply the output by -1.

$= -(|x + 3| + 1)$ — Substitute $|x + 3| + 1$ for $f(x)$.

$= -|x + 3| - 1$ — Distributive Property

The reflected function is $g(x) = -|x + 3| - 1$.

b. A reflection in the y-axis changes the sign of each input value.

$h(x) = f(-x)$ — Multiply the input by -1.

$= |-x + 3| + 1$ — Replace x with $-x$ in $f(x)$.

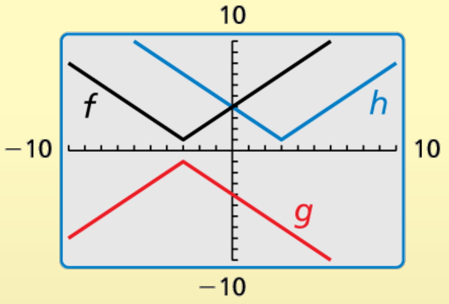
$= |-(x - 3)| + 1$ — Factor out -1.

$= |-1| \cdot |x - 3| + 1$ — Product Property of Absolute Value

$= |x - 3| + 1$ — Simplify.

The reflected function is $h(x) = |x - 3| + 1$.

Check



Homework: pg 16 #3-16, 33, 37-40

And remember:

- Horizontal translation (need to adjust every x): $y = f(x - h)$
- Vertical translation (need to adjust every y): $y = f(x) + k$
- Reflection in x-axis (need to negate every y): $y = -f(x)$
- Reflection in y-axis (need to negate every x): $y = f(-x)$